Worksheet Ten **MATH 104** Name:____________________

WORK ON THIS ASSIGNMENT IN GROUP OF 2-4. TURN IN YOUR WORK INDIVIDUALLY IN CLASS. YOU CAN USE YOUR NOTES FOR THIS ASSIGNMENT.

8.3: Polar Coordinates

- Polar Versus Rectangular (Cartesian) Coordinates

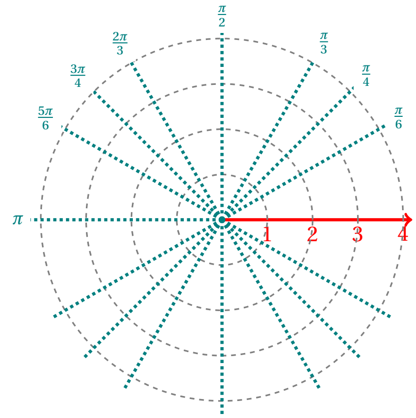
  Points in rectangular coordinates, (x, y) get plotted on the rectangular grids. Points in polar coordinates, (r,θ) get plotted on polar grids.

  Rectangular coordinates shows the distances from the x and y axis. Polar coordinates are involved with the distance from origin and the angle made with x-axis.

- The Basics:
  - In polar coordinates, a point is a distance and direction from the origin.
  - The polar axis is the half-line starting at origin.
  - The notation is $P(r,\theta)$ for $r > 0$, r is the distance between the point and origin. θ is the angle between the polar axis and the segment $\bar{OP}$, where O is the origin.
  - If $r < 0$ then $P(r,\theta) = P(|r|,\theta+\pi)$.
  - A point $P(r,\theta)$ can also be represented by $P(r,\theta+2k\pi)$ for any integer k.
  - The Grid: A grid for polar coordinates consists of circles which are used to mark r and lines through origin which are used to mark θ.

- Plotting the Points:

  Find the ray $\theta = \theta_0$ and circle $r = r_0$ the intersection is point (r_0, θ_0).



- Converting from Polar coordinates to Cartesian: $(r, \theta) \to (x, y)$

  $x = r\cos\theta$ and $y = r\sin\theta$

  Now, you can complete Problem 1.

- Converting From Cartesian to Polar coordinates. $(x, y) \to (r, \theta)$

  Use the formula $r^2 = x^2 + y^2$ and $\tan(\theta) = \dfrac{y}{x}$, $x \neq 0$, $y \neq 0$. To find the θ, use plotting.

  If x or y is zero, **ONLY** use plotting to find the coordinates.

  When $x \neq 0$ and $y \neq 0$, a simple rule of thumb is that if we choose $r = \sqrt{x^2 + y^2} > 0$, then one choice is

$$\begin{cases} \text{If } x > 0, & \theta = \arctan\left(\dfrac{y}{x}\right) \\[2ex] \text{If } x < 0, & \theta = \arctan\left(\dfrac{y}{x}\right) + \pi \end{cases}$$

  Now, you can complete Problem 2.

- Converting Equations From Cartesian Coordinate to Polar Coordinates:

  Replace x by $r\cos(\theta)$ and y by $r\sin(\theta)$.

- Converting Equations From Polar Coordinate to Cartesian Coordinates:

  Replace r and θ using $r = \sqrt{x^2 + y^2}$, $\sin(\theta) = \dfrac{y}{\sqrt{x^2 + y^2}}$, $\cos(\theta) = \dfrac{x}{\sqrt{x^2 + y^2}}$ and $\tan(\theta) = \dfrac{y}{x}$.

  Sometimes there are more straightforward ways:

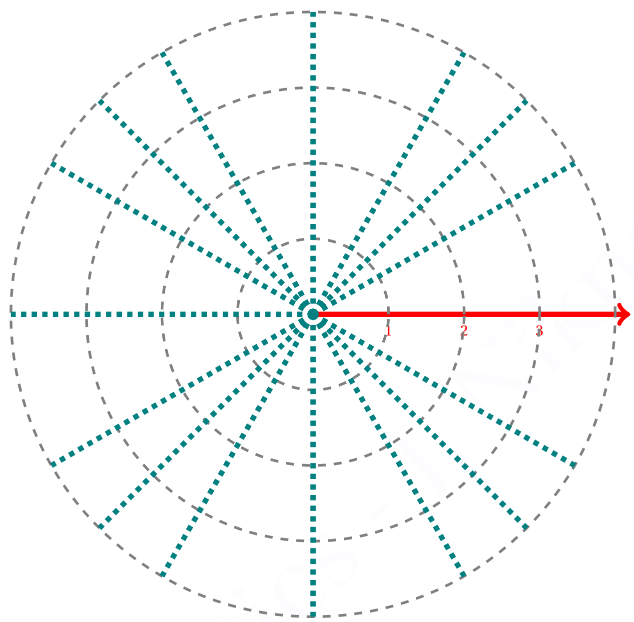
  Try to covert all into r^2, $r\cos(\theta)$, $r\sin(\theta)$... to convert r and θ.

  Find other ways to convert the r and θ.

  Now, you can complete Problems 3 and 4.

1. Plot and convert each of the following points in polar coordinates to rectangular coordinates.

   (a) $A(2\sqrt{2}, 5\pi/4)$
   (b) $B(2\sqrt{2}, -\pi/4)$
   (c) $C(4, 5\pi/4)$
   (d) $D(-4, \pi/4)$
   (e) $E(-2, \pi/4)$
   (f) $F(-3, 5\pi/3)$
   (g) $G(3, 2\pi/3)$
   (h) $H(3, 8\pi/3)$
   (i) $I(\sqrt{3}, \pi/3)$



2. Find polar coordinates for the point whose rectangular coordinates are

(a) $(-2,-2)$

(b) $(2,-2)$

(c) $(-2,2)$

(d) $(\sqrt{3},2)$

(e) $(3,3)$

(f) $(2,3)$

(g) $(2,0)$

(h) $(0,-2)$

3. Convert the following Cartesian equations to polar coordinates equations.

   (a) $x^2 + y^2 = 16$

   (b) $y = 5$

   (c) $x^2 - y^2 = 3$

4. Convert the following polar coordinates to rectangular coordinates equations. (Here a is a constant.)

   (a) $r = 3$

   (b) $r = 9\sin(\theta)$

   (c) $r = 2\sec(\theta)$

   (d) $\theta = \frac{\pi}{4}$

   (e) $r = 2 + \cos(\theta)$

   (f) $r = a$

   (g) $r = 2a\cos(\theta)$

   (h) $r = 2a\sin(\theta)$

Example Videos:

1. https://mediahub.ku.edu/media/t/1_apm1f0od